

# A note on one-way quantum deficit and quantum discord

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**Abstract** One-way quantum deficit and quantum discord are two important measures of quantum correlations. We revisit the relationship between them in two-qubit systems. We investigate the conditions that both one-way quantum deficit and quantum discord have the same optimal measurement ensembles, and demonstrate that one-way quantum deficit can be derived from the quantum discord for a class of  $X$  states. Moreover, we give an explicit relation between one-way quantum deficit and entanglement of formation. We show that under phase damping channel both one-way quantum deficit and quantum discord evolve exactly in the same way for four parameter  $X$  states. Some examples are presented in details.

**Keywords** One-way quantum deficit, Quantum discord, Entanglement of formation

## 1 Introduction

Quantum entanglement plays important roles in quantum information and quantum computation [1]. However, some quantum states without quantum entanglement can also perform quantum tasks [2, 3] like quantum state discrimination [4, 5], remote state preparation [6], quantum state merging [7, 8] etc., which have led to new definitions of quantum correlations such as quantum discord [9, 10], one-way quantum deficit [11–14], and various ‘discord like’ measures [15].

One-way quantum deficit was first proposed by Oppenheim et al [11] for studying thermodynamical systems. They considered the amount of work which could be extracted from a heat bath by local operations. It quantifies the minimum distillable entanglement generated between the whole system and the measurement apparatus in measuring one subsystem of the whole system [16]. The analytical formulae of one-way quantum deficit is not known even for two-qubit states. With limited analytical results [17, 18], many discussions on quantum deficit only rely on numerical results, since it involves minimization of sum of local and conditional entropies.

Another famous measure of quantum correlations, the quantum discord [9, 10], is defined to be the difference of two classically equivalent expressions for the mutual information. There have been a lot of results on quantum discord for bipartite as well as multipartite mixed quantum states [15]. Nevertheless, due to the optimization problem involved, it has been recently shown that calculating quantum discord is an NP complexity problem [19].

It is meaningful to link directly one-way quantum deficit to quantum discord. The relationship between quantum discord and one-way quantum deficit was first discussed in Ref. [14]. Horodecki *et al* shew that the one-way quantum deficit is upper bounded by the quantum discord for any bipartite quantum states. In Ref. [20] a tradeoff relationship between one-way unlocalizable quantum discord and one-way unlocalizable quantum deficit has been presented. The tradeoff relationship between quantum discord and one-way quantum deficit is obtained [21].

Anyway, decisive results between quantum discord and one-way quantum deficit is not fully explored even for the two-qubit  $X$  states yet. Here, we revisit the relationship between one-way quantum deficit and quantum discord. We find that for special two-qubit  $X$  states the one-way quantum deficit can be derived from quantum discord exactly in some optimal measurement bases. Furthermore, we connect one-way quantum deficit to entanglement of formation directly.

To capture the non-classical correlations in bipartite systems, let us recall the following two popular measures of quantum correlations.

*One-way quantum deficit* Suppose Alice and Bob are allowed to perform only local operations. Consider a one-way classical communication, say, from Alice to Bob. The amount of information extractable from quantum system  $\varrho^{AB}$  is given by  $\mathcal{I}_e = \log_2 \mathcal{D} - S(\varrho^{AB})$ , where  $\mathcal{D}$  is the dimension of the Hilbert space,  $S(\varrho) = -\text{Tr}[\varrho \log_2 \varrho]$  is the von Neumann entropy of a quantum state  $\varrho$ .

The classical operations to extract the amount of information from the quantum state is  $\mathcal{I}_o = \log_2 \mathcal{D} - \min S((\varrho^{AB})')$ , where  $(\varrho^{AB})' = \sum_k M_k^A \varrho^{AB} M_k^A$  is the quantum state after measurement  $M_k^A$  has been performed on  $A$ . The one-way quantum deficit [11–14] is given by the difference of  $\mathcal{I}_e$  and  $\mathcal{I}_o$  [16],

$$\begin{aligned} \vec{\Delta} &= \mathcal{I}_e - \mathcal{I}_o \\ &= \min_k S(\sum_k M_k^A \varrho^{AB} M_k^A) - S(\varrho^{AB}). \end{aligned} \tag{1}$$

The minimum is taken over all local measurements  $M_k^A$ . This quantity is equal to the thermal discord [22].

*Quantum discord* The quantum discord is defined as the minimal difference between quantum mutual information and classical correlation. The quantum mutual information is denoted by  $\mathcal{I}(\varrho^{AB}) = S(\varrho^A) + S(\varrho^B) - S(\varrho^{AB})$ , which is also identified as the total correlation of the bipartite quantum system  $\varrho^{AB}$ . The  $\varrho^{A(B)}$  are the reduced density matrices  $\text{Tr}_{B(A)}\varrho^{AB}$ , respectively. Let  $\{M_k^A\}$  be a measurement on subsystem  $A$ . Classical correlation is given as  $\mathcal{J}(\varrho^{AB}) = S(\varrho^B) - \min \sum_k p_k S(\varrho_{M_k^A}^B)$ , where  $p_k = \text{Tr}(M_k^A \otimes I_2 \varrho^{AB})$  is the probability of  $k$ th measurement outcome and  $\varrho_{M_k^A}^B = \text{Tr}_A[M_k^A \otimes I_2 \varrho^{AB}]/p_k$  is the post-measurement state.

The quantum discord [9, 10] is defined by

$$\begin{aligned} \vec{\delta} &= \mathcal{I}(\varrho^{AB}) - \mathcal{J}(\varrho^{AB}) \\ &= S(\varrho^A) + \min \sum_k p_k S(\varrho_{M_k^A}^B) - S(\varrho^{AB}). \end{aligned} \quad (2)$$

The superscript “ $\rightarrow$ ” stands for that the measurement performed on subsystem  $A$ . The minimum is taken over all possible measurements  $\{M_k^A\}$  on the subsystem  $A$ .

## 2 Linking one-way quantum deficit to quantum discord

Let us consider bipartite systems in Hilbert space  $\mathbb{C}^2 \otimes \mathbb{C}^2$ . Generally, the quantum correlations are invariant under local unitary operations [15]. Hence, one can write the  $X$  states [23] in the form

$$\varrho^{AB} = \frac{1}{4}(I_2 \otimes I_2 + a\sigma_z \otimes I_2 + bI_2 \otimes \sigma_z + \sum_{i \in \{x,y,z\}} c_i \sigma_i \otimes \sigma_i), \quad (3)$$

where  $\sigma_i (i \in \{x, y, z\})$  are Pauli matrices,  $I_2$  is the identity matrix, and the parameters  $\{a, b, c_x, c_y, c_z\} \in [-1, 1]$  are real numbers.

The optimal measurement with measurement operators satisfying  $M_k^A \geq 0$ ,  $\sum_k M_k^A = I$ , are generally positive operator-valued measurement (POVM). For rank-two two-qubit systems, the optimal measurement is just projective ones [24]. It is also sufficient to consider projective measurement for rank-three and four [25].

Let  $M_k^A = |k'\rangle\langle k'|$ ,  $k \in \{0, 1\}$ , where

$$|0'\rangle = \cos(\theta/2)|0\rangle - e^{-i\phi} \sin(\theta/2)|1\rangle, \quad (4)$$

$$|1'\rangle = e^{i\phi} \sin(\theta/2)|0\rangle + \cos(\theta/2)|1\rangle. \quad (5)$$

For the given system (3), we obtain  $\bar{\delta} = S(\varrho^A) + \min \sum_k p_k S(\varrho_{M_k^A}^B) - S(\varrho^{AB})$ , in which

$$p_{k \in \{0,1\}} = \frac{1}{2}(1 \pm a \cos \theta), \quad (6)$$

$S(\varrho^A) = h(\frac{1+a}{2})$  with  $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ ,

$$\begin{aligned} \sum_k p_k S(\varrho_{M_k^A}^B) &= p_0 S(\varrho_{M_0^A}^B) + p_1 S(\varrho_{M_1^A}^B) \\ &= - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log w_{kj}, \end{aligned} \quad (7)$$

with  $w_{00}, w_{01}$  and  $w_{10}, w_{11}$  the eigenvalues of  $\varrho_{M_0^A}^B$  and  $\varrho_{M_1^A}^B$ , respectively,

$$w_{kj \in \{0,1\}} = \{1 + (-1)^k a \cos \theta + (-1)^j \sqrt{[c_x^2 \cos^2 \phi + c_y^2 \sin^2 \phi] \sin^2 \theta + [b + (-1)^k c_z \cos \theta]^2}\} / (4p_k). \quad (8)$$

The corresponding quantity  $S(\sum_k M_k^A \varrho^{AB} M_k^A)$  in the definition of one-way quantum deficit is given by

$$\begin{aligned} S(\sum_k M_k^A \varrho^{AB} M_k^A) &= S(M_0^A \otimes p_0 \varrho_{M_0^A}^B + M_1^A \otimes p_1 \varrho_{M_1^A}^B) \\ &= S(p_0 \varrho_{M_0^A}^B) + S(p_1 \varrho_{M_1^A}^B) \\ &= - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log p_k w_{kj} \\ &= h(p_0) - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log w_{kj}. \end{aligned} \quad (9)$$

Substituting Eq. (7) into above equation, we have

$$S(\sum_k M_k^A \varrho^{AB} M_k^A) = h(p_0) + \sum_k p_k S(\varrho_{M_k^A}^B), \quad (10)$$

which is joint entropy theorem [26].

Let us set

$$\mathcal{F} = S(\varrho^A) + \sum_k p_k S(\varrho_{M_k^A}^B) - S(\varrho^{AB}), \quad (11)$$

$$\mathcal{G} = S(\sum_k M_k^A \varrho^{AB} M_k^A) - S(\varrho^{AB}). \quad (12)$$

Inserting Eq.(7), Eq.(9) into Eq.(11), Eq.(12), respectively, we have

$$\mathcal{F} = S(\varrho^A) - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log w_{kj} - S(\varrho^{AB}), \quad (13)$$

$$\mathcal{G} = h(p_0) - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log w_{kj} - S(\varrho^{AB}). \quad (14)$$

To search for the minimization involved in computing quantum discord and one-way quantum deficit is equivalent to seek for the minimal value of the function  $\mathcal{F}$  and  $\mathcal{G}$  with respect to the two parameters  $\theta$  and  $\phi$  in the measurement operators. Similar to the technique to minimize  $\mathcal{F}(\theta, \phi)$ , it is enough to consider minimize  $\mathcal{F}(\theta, 0)$  in calculating the quantum discord of two-qubit  $X$ -states [27]. We denote

$$G(\theta, \phi) = S\left(\sum_k M_k^A \varrho^{AB} M_k^A\right) = - \sum_{k,j \in \{0,1\}} p_k w_{kj} \log p_k w_{kj} = - \sum_{l=1}^4 \lambda_l \log_2 \lambda_l,$$

where,

$$\lambda_{1,2} = \frac{1}{4} \left( p_0 \pm \sqrt{R + T_0} \right), \quad \lambda_{3,4} = \frac{1}{4} \left( p_1 \pm \sqrt{R + T_1} \right),$$

and  $p_0 = 1 + a \cos \theta$ ,  $p_1 = 1 - a \cos \theta$ ,  $R = [c_x^2 \cos^2 \phi + c_y^2 \sin^2 \phi] \sin^2 \theta$ ,  $T_0 = (b + c_z \cos \theta)^2$ ,  $T_1 = (b - c_z \cos \theta)^2$ . Since  $\lambda_l \geq 0$ , one has  $p_k \geq \sqrt{R + T_k} \geq 0$ .

Noting that  $G(\theta, \phi) = G(\pi - \theta, \phi) = G(\theta, 2\pi - \phi)$  and  $G(\theta, \phi)$  is symmetric with respect to  $\theta = \pi/2$  and  $\phi = \pi$ , we only need to consider the case of  $\theta \in [0, \pi/2]$  and  $\phi \in [0, \pi)$ . The extreme points of  $G(\theta, \phi)$  are determined by the first partial derivatives of  $G$  with respect to  $\theta$  and  $\phi$ ,

$$\frac{\partial G}{\partial \theta} = -\frac{\sin \theta}{4} H_\theta, \quad (15)$$

with

$$\begin{aligned} H_\theta = & \frac{R \csc \theta \cot \theta - c_z \sqrt{T_0}}{\sqrt{R + T_0}} \log_2 \frac{p_0 + \sqrt{R + T_0}}{p_0 - \sqrt{R + T_0}} + a \log_2 \frac{p_1^2 - (R + T_1)}{p_0^2 - (R + T_0)} \\ & + \frac{R \csc \theta \cot \theta + c_z \sqrt{T_1}}{\sqrt{R + T_1}} \log_2 \frac{p_1 + \sqrt{R + T_1}}{p_1 - \sqrt{R + T_1}}, \end{aligned} \quad (16)$$

and

$$\frac{\partial G}{\partial \phi} = 2 e f \sin^2 \theta \sin 2\phi H_\phi, \quad (17)$$

with

$$H_\phi = \frac{1}{\sqrt{R + T_0}} \log_2 \frac{p_0 + \sqrt{R + T_0}}{p_0 - \sqrt{R + T_0}} + \frac{1}{\sqrt{R + T_1}} \log_2 \frac{p_1 + \sqrt{R + T_1}}{p_1 - \sqrt{R + T_1}}, \quad (18)$$

$e = \frac{1}{4} |c_x + c_y|$  and  $f = \frac{1}{4} |c_x - c_y|$  where the absolute values have been taken since the phase for  $X$  states can be always removed by local unitary operation [15].

As  $H_\phi$  is always positive,  $\frac{\partial G}{\partial \phi} = 0$  implies that either  $\phi = 0, \pi/2$  for any  $\theta$ , or  $\theta = 0$  for any  $\phi$  which implies that Eq. (15) is zero and the minimization is independent on  $\phi$ . If  $\theta \neq 0$ , one gets the second derivative of  $G$ ,

$$\left. \frac{\partial^2 G}{\partial \phi^2} \right|_{(\theta, 0)} = 4ef \sin^2(\theta) H_{\phi=0} > 0,$$

and

$$\left. \frac{\partial^2 G}{\partial \phi^2} \right|_{(\theta, \pi/2)} = -4ef \sin^2(\theta) H_{\phi=\pi/2} < 0.$$

Since for any  $\theta$  the second derivative  $\partial^2 G / \partial \phi^2$  is always negative for  $\phi = \pi/2$ , we only need to deal with the minimization problem for the case of  $\phi = 0$ . To minimize  $G(\theta, \phi)$  becomes to minimize  $G(\theta, 0)$ . Thus, we need only to find the minimal value of  $\mathcal{F}$  and  $\mathcal{G}$  by varying  $\theta$  only.

Denote  $\mathcal{F}(\theta) = \mathcal{F}|_{\phi=0}$ ,  $\mathcal{G}(\theta) = \mathcal{G}|_{\phi=0}$  and  $\mathcal{H}(\theta) = \mathcal{G}(\theta) - \mathcal{F}(\theta)$ . The first derivative of  $\mathcal{H}(\theta)$  with respect to  $\theta$  is given by

$$\mathcal{H}(\theta)' = \frac{a}{2} \sin \theta \log_2 \frac{1 + a \cos \theta}{1 - a \cos \theta}. \quad (19)$$

From  $\mathcal{H}(\theta)' = 0$ , we have either  $a = 0$  or  $\theta = 0, \pi/2$ . Since these stationary points make  $\mathcal{F}(\theta)' = \mathcal{G}(\theta)'$ , they are the sufficient conditions that both  $\mathcal{G}(\theta)$  and  $\mathcal{F}(\theta)$  reach the minimum with the same optimal measurement ensemble. Here  $a$  is a parameter of the  $X$  states and  $\theta$  is a parameter related to measurement. Substituting  $a = 0$  or  $\theta = 0, \pi/2$  into  $\mathcal{F}(\theta)$  and  $\mathcal{G}(\theta)$  we have the following results:

*Theorem* For two-qubit  $X$  states, if the measurement is performed on the subsystem  $A$  (resp.  $B$ ), then  $\vec{\Delta} = \vec{\delta}$  for  $a = 0$  (resp.  $b = 0$ ). If their optimal measurement bases which depend on the parameters of the state are at  $\theta = 0$ , then  $\vec{\Delta} = \vec{\delta}$ . If their optimal measurement bases which depend on the parameters of the state are at  $\theta = \pi/2$ , then  $\vec{\Delta} = \vec{\delta} - S(\varrho^A) + 1$ .

Recently, we notice that in Ref [28], the authors assumed that the quantum discord and one-way quantum deficit get their minimal values in the same measurement ensemble simultaneously. Thus similar to the quantum discord, the frozen quantum phenomenon under bit flip channels of one-way quantum deficit happens. Here, our *Theorem* gives the explicit conditions that both quantum discord and one-way quantum deficit have the same optimal measurement bases.

*Corollary 1* The one-way quantum deficit is bounded by the quantum discord for two-qubit  $X$  states,

$$\vec{\delta} \leq \vec{\Delta} \leq S(\rho^A). \quad (20)$$

*Proof.* Since  $0 \leq \mathcal{H} \leq 1$ , we have  $\vec{\delta} \leq \vec{\Delta} \leq \vec{\delta} + 1$ . By using the tight bound about one-way quantum deficit  $\Delta \leq S(\rho^A)$  in Ref. [26], we obtain (20). ■

*Corollary 2* One-way quantum deficit and the entanglement of formation satisfy the following relations for two-qubit  $X$  states,

$$\vec{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + \begin{cases} 1, & a=0 \text{ or } \theta = \pi/2; \\ h(\frac{1-a}{2}), & \theta = 0, \end{cases} \quad (21)$$

where  $C$  is the assisted system to purify the state  $\varrho^{AB}$ , and  $E_f(\varrho^{BC})$  is the entanglement of formation of  $\varrho^{BC}$ , while  $\varrho^{BC}$  is the reduced state from a pure state  $|\psi\rangle_{ABC}$ .

*Proof.* From the Koashi-Winter equality [29]

$$S(\varrho^B) = \mathcal{J}(\varrho^{AB}) + E_f(\varrho^{BC}), \quad (22)$$

and  $\mathcal{J}(\varrho^{AB}) = S(\varrho^B) - \min \sum_k p_k S(\varrho_{M_k^A}^B)$ , one has  $E_f(\varrho^{BC}) = \min \sum_k p_k S(\varrho_{M_k^A}^B)$ . Consequently, quantum discord is rewritten as

$$\vec{\delta} = S(\varrho^A) + E_f(\varrho^{BC}) - S(\varrho^{AB}). \quad (23)$$

Thus, we have

$$\vec{\Delta} = \vec{\delta} = h(\frac{1-a}{2}) + E_f(\varrho^{BC}) - S(\varrho^{AB}), \quad (24)$$

where both of the optimal measurement bases are taken at  $\theta = 0$ . Hence for  $a = 0$ , we have  $\vec{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1$  indeed. For  $\theta = \pi/2$ , by using the relations in *Theorem* and Eq.(23) we also get  $\vec{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1$ . ■

*Remark* Recently, in Ref. [21] by using measure of relative entropy of coherence,

$$C_{RE}(\varrho^A) = \min_{\sigma \in \mathcal{I}} S(\varrho^A || \sigma), \quad (25)$$

where  $\mathcal{I}$  stands for the set of decoherence states  $\sigma = \sum_i \mu_i |i\rangle\langle i|$  with  $\mu_i \in [0, 1]$  and  $\sum_i \mu_i = 1$ , the authors provided a tradeoff relationship between  $\vec{\delta}$  and  $\vec{\Delta}$ , i. e.,  $\vec{\delta} + C_{RE}(\varrho^A) = \vec{\Delta}$ .

In fact, one-way quantum deficit can be derived from quantum discord directly. We consider the exact relationship between quantum discord and one-way quantum deficit in the following examples.

*Example 1.* The Bell-diagonal state  $\varrho_{Bell}^{AB} = \frac{1}{4}(I_2 \otimes I_2 + \sum_{i \in \{x,y,z\}} c_i \sigma_i \otimes \sigma_i)$ . In this case  $a = 0$  and

$$\vec{\Delta} = \vec{\delta} = h\left(\frac{1-c}{2}\right) + \sum_{s \in \{jkl\}} A_s \log_2 A_s, \quad (26)$$

where  $s$  is the set  $\{jkl\} = \{111, 100, 010, 001\}$ ,  $A_{jkl} = \frac{1}{4}(1 + (-1)^j c_x + (-1)^k c_y + (-1)^l c_z)$ , and  $c \equiv \max\{|c_x|, |c_y|, |c_z|\}$ . Therefore, from *Theorem* we get the analytical expression of one-way quantum deficit from quantum discord given in [30].

*Example 2.* Consider a class of X-state,

$$\varrho_q^{AB} = q|\psi^-\rangle\langle\psi^-| + (1-q)|00\rangle\langle 00|, \quad (27)$$

where  $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ .

For this state, quantum discord is derived at  $\theta = \pi/2$  for  $q \in [0, 1]$ . The optimal basis of one-way quantum deficit for  $q \in [0.67, 1]$  is also at  $\theta = \pi/2$ . The value 0.67 is the solution of  $H'_\theta|_{\theta=\pi/2, \phi=0} = 0$  in Eq. (16) for the state  $\varrho_q^{AB}$ . According to the *Theorem*, we have

$$\vec{\Delta} = E_f(\varrho^{BC}) - S(\varrho^{AB}) + 1, \quad (28)$$

where the entanglement of formation

$$E_f(\varrho^{BC}) = h\left(\frac{1 + \sqrt{1 - \mathcal{C}^2}}{2}\right) \quad (29)$$

with concurrence  $\mathcal{C} = \sqrt{2q(1-q)}$ . So analytical one-way quantum deficit of state  $\varrho_q^{AB}$  is

$$\vec{\Delta} = h\left(\frac{1 + \sqrt{1 - \mathcal{C}^2}}{2}\right) - h(q) + 1 \quad (30)$$

for  $q \in [0.67, 1]$ , see Fig. 1.

### 3 Quantum correlations under phase damping channel

A quantum system would be subject to interaction with environments. We consider now the evolution of one-way quantum deficit and quantum discord under noisy channels. Consider a class of initial two-qubit states,

$$\Omega = \frac{1}{4}(I_2 \otimes I_2 + bI_2 \otimes \sigma_z + \sum_{i \in \{x,y,z\}} c_i \sigma_i \otimes \sigma_i). \quad (31)$$



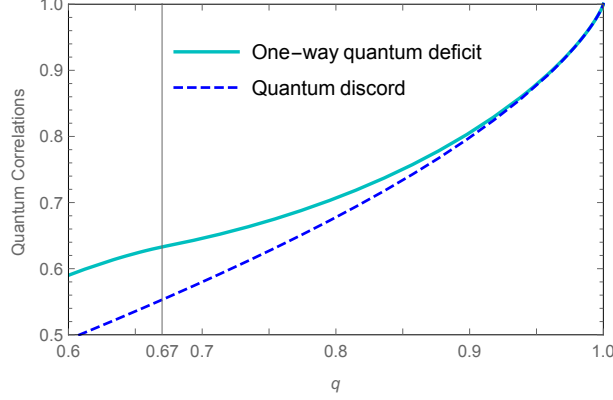


FIG. 1: One-way quantum deficit (turquoise solid line) and quantum discord (blue dashed line) vs  $q$ . The interval  $q \in [0.67, 1]$  one-way quantum deficit and quantum discord both get their optimum at  $\theta = \pi/2$ .

If both two qubits independently goes through a channel given by the Kraus operators  $\{K_i\}$ ,  $\sum_i K_i^\dagger K_i = I$ . The state  $\Omega$  evolves into

$$\tilde{\Omega} = \sum_{i,j \in \{1,2\}} K_i^A \otimes K_j^B \cdot \Omega \cdot [K_i^A \otimes K_j^B]^\dagger. \quad (32)$$

For phase damping channels [31], the Kraus operators are given by  $K_1^{A(B)} = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$ , and  $K_2^{A(B)} = \sqrt{\gamma}|1\rangle\langle 1|$  with the decoherence rate  $\gamma \in [0, 1]$ . Thus we have

$$\tilde{\Omega} = \frac{1}{4}[I_2 \otimes I_2 + bI_2 \otimes \sigma_z + c_z \sigma_z \otimes \sigma_z + \sum_{i \in \{x,y\}} (1-\gamma)c_i \sigma_i \otimes \sigma_i], \quad (33)$$

which is a two-qubit  $X$  state with  $a = 0$ . From the *Theorem*, we obtain one-way quantum deficit and quantum discord performed on the subsystem  $A$  evolves coincidentally with each other all the time.

For example, we draw the quantum discord and one-way quantum deficit vs parameter  $\gamma$  in Fig. 2 for  $b = 0.26$ ,  $c_x = 0.13$ ,  $c_z = 0.08$ , and  $c_y = 0.15, 0.25, 0.35, 0.45, 0.55$  respectively.

#### 4 Conclusions

We have investigated the connections between one-way quantum deficit and quantum discord for two-qubit  $X$  states. Sufficient conditions are given that the one-way quantum deficit can be derived from quantum discord directly. The explicit relation between one-way quantum deficit and entanglement of formation is also presented. Moreover, we have shown that the one-way quantum deficit and quantum discord of a class of four parameters  $X$

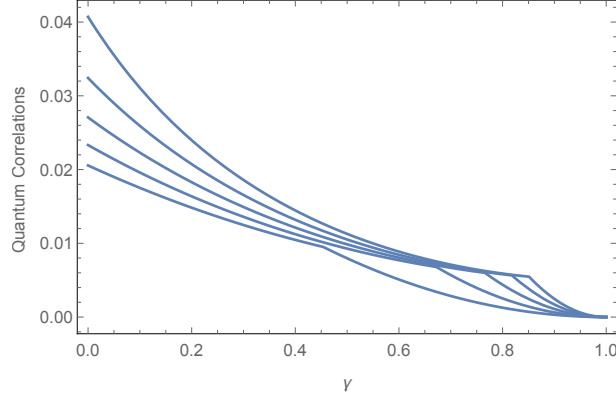


FIG. 2: One-way quantum deficit and quantum discord evolve exactly in the same way under phase damping channel. The solid line from bottom to top correspond  $c_y = 0.15, 0.25, 0.35, 0.45, 0.55$  respectively, for fixed parameters  $b = 0.26, c_x = 0.13$ , and  $c_z = 0.08$ .

states evolve coincidentally under phase damping channel. Our results may enlighten the understanding on the relations between one-way quantum deficit and quantum discord. It is also interesting to study the relationship between one-way quantum deficit and quantum discord for higher dimensional and multipartite systems.

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1. Horodecki, R., Horodecki, P., Horodecki, M., Horodecki, K.: Quantum entanglement. Rev. Mod. Phys. **81**, 865 (2009)
  2. Datta, A., Shaji, A., Caves, C.M.: Quantum discord and the power of one qubit. Phys. Rev. Lett. **100**, 050502 (2008)
  3. Lanyon, B.P., Barbieri, M., Almeida, M.P., White, A.G.: Experimental quantum computing without entanglement. Phys. Rev. Lett. **101**, 200501 (2008)
  4. Roa, L., Retamal, J.C., Alid-Vaccarezza, M.: Dissonance is required for assisted optimal state discrimination. Phys. Rev. Lett. **107**, 080401 (2011)
  5. Li, B., Fei, S.M., Wang, Z.X., Fan, H.: Assisted state discrimination without entanglement.

- Phys. Rev. A **85**, 022328 (2012)
6. Dakić, B., Lipp, Y.O., Ma, X., Ringbauer, M., Kropatschek, S., Barz, S., Paterek, T., Vedral, V., Zeilinger, A., Brukner, Č., et al.: Quantum discord as resource for remote state preparation. Nat. Phys. **8**, 666 (2012)
  7. Madhok, V., Datta, A.: Interpreting quantum discord through quantum state merging. Phys. Rev. A **83**, 032323 (2011)
  8. Cavalcanti, D., Aolita, L., Boixo, S., Modi, K., Piani, M., Winter, A.: Operational interpretations of quantum discord. Phys. Rev. A **83**, 032324 (2011)
  9. Ollivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. **88**, 017901 (2001)
  10. Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A: Math. Gen. **34**, 6899 (2001)
  11. Oppenheim, J., Horodecki, M., Horodecki, P., Horodecki, R.: Thermodynamical approach to quantifying quantum correlations. Phys. Rev. Lett. **89**, 180402 (2002)
  12. Horodecki, M., Horodecki, K., Horodecki, P., Horodecki, R., Oppenheim, J., Sen(De), A., Sen, U.: Local information as a resource in distributed quantum systems. Phys. Rev. Lett. **90**, 100402 (2003)
  13. Devetak, I.: Distillation of local purity from quantum states. Phys. Rev. A **71**, 062303 (2005)
  14. Horodecki, M., Horodecki, P., Horodecki, R., Oppenheim, J., Sen(De), A., Sen, U., Synak-Radtke, B.: Local versus nonlocal information in quantum-information theory: formalism and phenomena. Phys. Rev. A **71**, 062307 (2005)
  15. Modi, K., Brodutch, A., Cable, H., Paterek, T., Vedral, V.: The classical-quantum boundary for correlations: discord and related measures. Rev. Mod. Phys. **84**, 1655 (2012)
  16. Streltsov, A., Kampermann, H., Bruß, D.: Linking quantum discord to entanglement in a measurement. Phys. Rev. Lett. **106**, 160401 (2011)
  17. Wang, Y.K., Ma, T., Li, B., Wang, Z.X.: One-way information deficit and geometry for a class of two-qubit states. Commun. Theor. Phys. **59**, 540 (2013)
  18. Wang, Y.K., Jing, N., Fei, S.M., Wang, Z.X., Cao, J.P., Fan, H.: One-way deficit of two-qubit  $X$  states. Quantum Inf. Process. **14**, 2487 (2015)
  19. Huang, Y.: Computing quantum discord is NP-complete. New J. Phys. **16**, 033027 (2014)
  20. Zhu, X.N., Fei, S.M.: One-way unlocalizable information deficit. J. Phys. A: Math. Theor. **46**,

- 325303 (2013)
21. Xi, Z., Li, Y., Fan, H.: Quantum coherence and correlations in quantum system. *Sci. Rep.* **5**, 10922 (2015)
  22. Zurek, W.H.: Quantum discord and Maxwell's demons. *Phys. Rev. A* **67**, 012320 (2003)
  23. Yurischev, M.: On the quantum discord of general  $X$  states. *Quantum Inf. Process.* **14**, 3399 (2015)
  24. Shi, M., Sun, C., Jiang, F., Yan, X., Du, J.: Optimal measurement for quantum discord of two-qubit states. *Phys. Rev. A* **85**, 064104 (2012)
  25. Galve, F., Giorgi, G.L., Zambrini, R.: Orthogonal measurements are almost sufficient for quantum discord of two qubits. *Europhys. Lett.* **96**, 40005 (2011)
  26. Shao, L.H., Xi, Z.J., Li, Y.M.: Remark on the one-way quantum deficit for general two-qubit states. *Commun. Theor. Phys.* **59**, 285 (2013)
  27. Maldonado-Trapp, A., Hu, A., Roa, L.: Analytical solutions and criteria for the quantum discord of two-qubit  $X$ -states. *Quantum Inf. Process.* **14**, 1947 (2015)
  28. Chanda, T., Pal, A.K., Biswas, A., Sen(De), A., Sen, U.: Freezing of quantum correlations under local decoherence. *Phys. Rev. A* **91**, 062119 (2015)
  29. Koashi, M., Winter, A.: Monogamy of quantum entanglement and other correlations. *Phys. Rev. A* **69**, 022309 (2004)
  30. Luo, S.: Quantum discord for two-qubit systems. *Phys. Rev. A* **77**, 042303 (2008)
  31. Nielsen, M.A., Chuang, I.L.: *Quantum Computation and Quantum Information*. Cambridge University Press (2010)